8-7: FACTORING SPECIAL CASES

Lesson Objectives:

- *Factoring perfect square trinomials* $(a \pm b)^2 = a^2 \pm 2ab + b^2$
- *Factoring the difference of two squares* $(a+b)(a-b) = a^2 - b^2$

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1. + at the end
2. quadratic term is a perfect square
3. constant term is a perfect square
4. linear term = 2ab

EXAMPLE 1: FACTORING A PERFECT SQUARE TRINOMIAL WITH a=1

Factor.

1. $x^2 + 8x + 16$ → $(x+4)^2$ (16 | +)
2. $x^2 + 24x + 144$ → $(x+12)^2$
3. $m^2 - 28m + 196$ → $(m-14)^2$
4. $a^2 + 26a + 169$ → $(a+13)^2$ (169 | + ; 1·169 ; 170)
5. $x^2 + 4x + 4$ → $(x+2)^2$
6. $g^2 - 2g + 1$ → $(g-1)^2$
7. $x^2 - 22x + 121$ → $(x-11)^2$
8. $x^2 + 25x + 100$ → $(x+5)(x+20)$ (100 | + ; 1·100 ; 2·50 ; 4·25 ; +5 +20 = +25)

EXAMPLE 2: FACTORING A PERFECT SQUARE TRINOMIAL WITH $a \neq 1$

Factor.

9. $4x^2 + 12x + 9$ → $(2x+3)^2$
10. $4x^2 - 20x + 25$ → $(2x-5)^2$
11. $9n^2 + 12n + 4$ → $(3n+2)^2$
12. $4x^2 - 60x + 225$ → $(2x-15)^2$
13. $9x^2 + 60x + 100$ → $(3x+10)^2$
14. $16m^2 - 72m + 81$ → $(4m-9)^2$
15. $16x^2 - 8x + 1$ → $(4x-1)^2$
16. $25b^2 + 60b - 36$ → Not a perfect square trinomial

17. $36x^2 - 84x + 49$

$(6x - 7)^2$

18. $9x^2 + 12x - 4$

Not a perfect square trinomial

19. $25x^4 + 40x^2y^2 + 16y^4$

$(5x^2 + 4y^2)^2$

20. $x^2 + 20x + 64$

$(x+4)(x+16)$

64 | +
1, 64
2, 32
+4, +16 20

1. Subtraction in the middle
2. First term is a perfect square
3. Last term is a perfect square

EXAMPLE 3: FACTORING THE DIFFERENCE OF TWO SQUARES FOR a=1

Factor.

21. $x^2 - 81$

$(x+9)(x-9)$

22. $x^2 - 64$

$(x+8)(x-8)$

23. $m^2 - 36$

$(m+6)(m-6)$

24. $x^2 + 9$

Prime

25. $x^2 - 100$

$(x+10)(x-10)$

26. $x^2 - 25$

$(x+5)(x-5)$

27. $m^2 - 225$

$(m+15)(m-15)$

28. $x^2 - 1$

$(x+1)(x-1)$

EXAMPLE 4: FACTORING THE DIFFERENCE OF TWO SQUARES FOR $a \neq 1$

Factor.

29. $16x^2 - 49$

$(4x+7)(4x-7)$

30. $4x^2 - 121$

$(2x+11)(2x-11)$

31. $9v^2 - 4$

$(3v+2)(3v-2)$

32. $25x^2 - 64$

$(5x+8)(5x-8)$

33. $4w^2 - 81$

$(2w+9)(2w-9)$

34. $100y^2 - 121$

$(10y-11)(10y+11)$

35. $4b^2 - 169$

$(2b+13)(2b-13)$

36. $16x^2 - 9$

$(4x+3)(4x-3)$

FACTORING COMPLETELY

1. GCF
2. SPECIAL CASES
   a) Perfect Square Trinomials
   b) Difference of Two Squares
3. TRINOMIALS
   a) $x^2 \pm bx \pm c$
   b) $ax^2 \pm bx \pm c$
4. FACTOR BY GROUPING
5. CAN I FACTOR AGAIN?

EXAMPLE 5: FACTORING OUT THE GCF FIRST
Factor completely.

37. $10x^2 - 40$

$10(x^2 - 4)$

$10(x+2)(x-2)$

38. $8y^2 - 50$

$2(4y^2 - 25)$

$2(2y+5)(2y-5)$

39. $3c^2 - 75$

$3(c^2 - 25)$

$3(c+5)(c-5)$

40. $28k^2 - 7$

$7(4k^2 - 1)$

$7(2k+1)(2k-1)$

Factor completely.

41. $4x^2 + 24x + 36$

$4(x^2 + 6x + 9)$

$4(x+3)^2$

$(2x+6)^2$

$(2x+6)(2x+6)$

$2(x+3)2(x+3)$

$4(x+3)^2$

42. $50x^2 - 162y^2$

$2(25x^2 - 81y^2)$

$2(5x+9y)(5x-9y)$

43. $50x^2 - 80x + 32$

$2(25x^2 - 40x + 16)$

$2(5x-4)^2$

44. $x^8 - 256$

$(x^4+16)(x^4-16)$

$(x^4+16)(x^2+4)(x^2-4)$

$(x^4+16)(x^2+4)(x+2)(x-2)$

Due Thu

Name _____ ***8-7 Practice Worksheet*** Period _____

Factor.

1. $h^2+12h+36$
2. $t^2-14t+49$
3. $100v^2-220v+121$
4. k^2-196
5. m^2-225
6. y^2-900
7. $9c^2-64$
8. $144p^2-1$
9. $25w^2-196$
10. $3x^2+48x+192$
11. $7h^2-56h+112$
12. $3m^2-12$
13. $6r^3-150r$
14. $16p^2-48pq+36q^2$
15. $28c^2+140cd+175d^2$
16. $x^2+x+\frac{1}{4}$
17. $64g^2-192gh+144h^2$
18. $\frac{1}{9}n^2-\frac{1}{25}$
19. $\frac{1}{25}k^2+\frac{6}{5}k+9$
20. $36m^4+84m^2+49$
21. $108m^6-147$
22. $x^{20}-4x^{10}y^5+4y^{10}$
23. $45x^4-60x^2y+20y^2$
24. $37g^8-37h^8$

The area of a square is given. Find the length of one of the sides of the square.

25. $49d^2+28d+4$

The expression $(t-3)^2-16$ is a difference of two squares.

26. Identify *a* and *b*.
27. Factor $(t-3)^2-16$.

The binomial $16-81n^4$ can be factored twice as the difference of two squares.

28. Factor $16-81n^4$ completely.
29. What characteristics do 16 and $81n^4$ share that make this possible?
30. Write a binomial that can be factored twice as the difference of two squares.

MIXED REVIEW

Is each number a solution of the given inequality (*not multiple choice; answer each letter yes/no*)?

31. $4z + 7 \geq 15$

a) -2 b) 2 c) 5

32. $-2g + 3 > 5$

a) -3 b) -1 c) 4

Solve each inequality. Graph the solution.

33. $z + 7 < 9$

34. $-5 + 4r \geq 3$

35. $5w > -6w + 11$

36. $4 + 3n \geq 1$ or $-5n > 25$

37. $10k < 75$ and $4 - k \leq 0$

38. $|4k - 2| = 11$